Name: ANSWERS

Instructor: Bullwinkle

Math 10120, Exam I.
October 14, 2014

- The Honor Code is in effect for this examination. All work is to be your own.
- Please turn off all cellphones and electronic devices.
- Calculators **are** allowed
- The exam lasts for 1 hour and 15 minutes.
- Be sure that your name and your instructor's name are on the front page of your exam.
- Be sure that you have all 16 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(•)	(d)	(e)
2.	(a)	(b)	(c)	(•)	(e)
3.	(a)	(b)	(c)	(d)	(•)
4.	(•)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(•)	(e)
6.	(a)	(b)	(c)	(•)	(e)
7.	(a)	(b)	(c)	(d)	(•)
8.	(a)	(•)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(•)
10.	(•)	(b)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice	
11.	
12.	
13.	
15.	
Total	

Multiple Choice

1.(5pts) Below is a population table for St. Joseph county and the three major population centers.

District	Population
Notre Dame	11,931
South Bend	100,800
Mishawaka	48,031
County	105,582
Total	266,344

A group of $1,500$ people from the community assemble for a "Take back the night" walk. Roughly how many would you expect to be from Notre Dame?

(a) 75 (b) 34 (c) 67 (d) 15 (e) 103

Solution. $\frac{11931 \cdot 1500}{266344} = 67.19317874628 \cdots$ so 67.

2.(5pts) If you flip a fair coin 5 times, what is the probability that you get exactly 3 heads?

(a) $\dfrac{P(5,3)}{5^2}$ (b) $\dfrac{P(5,3)}{2^5}$ (c) $\dfrac{C(5,3)}{P(5,3)}$

(d) $\dfrac{C(5,3)}{2^5}$ (e) $\dfrac{C(5,3)}{5^2}$

Solution. The number of sequences with 3 heads is $C(5,3)$ since you just need to tell me where to put the heads. The number of all sequences is 2^5 so $\dfrac{C(5,3)}{2^5}$.

3.(5pts) A sample space consists of 7 simple outcomes $\{a, b, c, d, e, f, g\}$. The probabilities are

P(a)	P(b)	P(c)	P(d)	P(e)	P(f)	P(g)
0.10	0.06	0.15	0.09	0.08	0.12	0.40

What is $\mathrm{P}(\{e, c, f\})$?

(a) 0.65 (b) $\frac{3}{7}$ (c) 0.00144 (d) 0.429 (e) 0.35

Solution. $0.08 + 0.15 + 0.12 = 0.35$

4.(5pts) Suppose $\mathrm{P}(E) = 0.4$, $\mathrm{P}(F) = 0.3$ and $\mathrm{P}(E \cap F) = 0.2$. Which of the statements below is correct?

(a) E and F are neither independent nor mutually exclusive.

(b) E and F are independent but not mutually exclusive.

(c) Neither Independence nor exclusivity can be determined from the given data.

(d) E and F are mutually exclusive but not independent.

(e) E and F are independent and mutually exclusive.

Solution. Since $\mathrm{P}(E \cap F) > 0$ the events are not mutually exclusive. Since $\mathrm{P}(E|F) = \dfrac{0.2}{0.4} = 0.5 \neq \mathrm{P}(F)$ the events are not independent.

5.(5pts) Suppose that E and F are events in an experiment, and $P(E) = \frac{1}{4}$, $P(F) = \frac{1}{2}$, $P(E \cup F) = \frac{3}{4}$. What is $\Pr(E|F)$.

(a) $\dfrac{1}{2}$ (b) $\dfrac{1}{4}$ (c) $\dfrac{1}{3}$ (d) 0 (e) 1

Solution.

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{P(E \cap F)}{1/2}.$$

$$P(E \cup F) = P(E) + P(F) - P(E \cap F) \quad \text{so} \quad \frac{3}{4} = \frac{1}{4} + \frac{1}{2} - P(E \cap F).$$

This gives:

$$P(E \cap F) = \frac{3}{4} - \frac{1}{4} - \frac{1}{2} = 0.$$

Thus $P(E|F) = 0$.

6.(5pts) A new piece of electronic equipment has five components. the probability of failure within a year is 0.1 for each component. Assuming that the failure of the various components are independent of each other, what is the probability that no component will fail in the first year?

(a) $(0.1)^5$ (b) 0.5 (c) $1 - (0.1)^5$

(d) $(0.9)^5$ (e) $1 - (0.9)^5$

Solution. We call the components Component 1, Component 2, Component 3, Component 4 and Component 5. We let Wi denote the probability that Component i is still working at the end of the first year. The probability that no component has failed in the first year is

$$P(W1 \cap W2 \cap W3 \cap W4 \cap W5) = P(W1)P(W2)P(W3)P(W4)P(W5)$$

by the assumption of independence.

$$P(W1)P(W2)P(W3)P(W4)P(W5) = (0.9)^5$$

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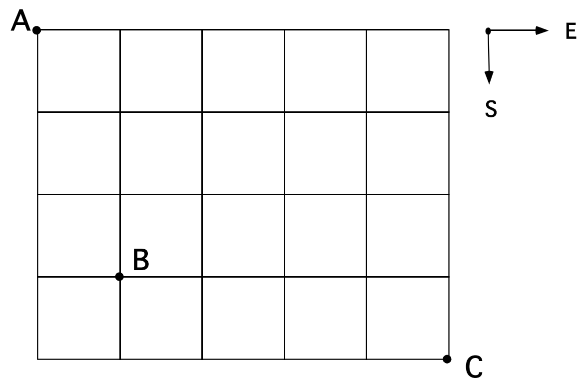
7.(5pts) A street map of Mathland is shown below. If a taxi driver chooses a random route from A to C traveling south and east only, what is the probability that he will **not pass through the intersection at B** ?



(a) $1 - \frac{4}{126}$ (b) $\frac{4}{126}$ (c) $1 - \frac{5}{126}$

(d) $\frac{20}{126}$ (e) $1 - \frac{20}{126}$

Solution. The probability he does not pass through B, $(P(B^c))$, is 1 minus the probability he will pass through B, $(P(B))$.

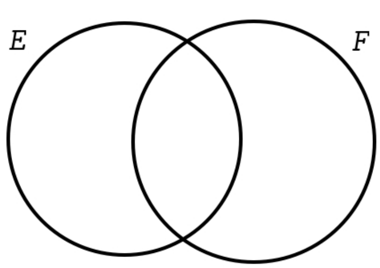
$$P(B) = \frac{\#\text{ admissible routes through B}}{\text{Total }\#\text{ of admissible routes}}$$

$$= \frac{(\#\text{ admissible routes from A to B})((\#\text{ admissible routes from B to C})}{\text{Total }\#\text{ of admissible routes}}$$

$$= \frac{C(4,1)\cdot C(5,4)}{C(9,5)} = \frac{4\cdot 5}{126} = \frac{20}{126}.$$

Thus $P(B^c) = 1 - \frac{20}{126}$.

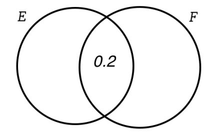
8.(5pts) Suppose given two events, E and F, such that $\mathrm{P}(E) = 40\%$, $\mathrm{P}(F) = 60\%$ and $\mathrm{P}(F|E) = 33\frac{1}{3}\%$. What is $\mathrm{P}\big((E \cup F)'\big)$?



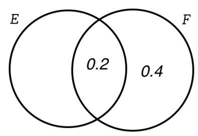
(a) 10% (b) 20% (c) 40% (d) 50% (e) 30%

 Initials: ____________

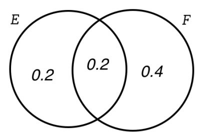
Solution. $P(F|E)$ is the probability that you are in F given that you are in E and $P(E \cap F) = P(F|E) \cdot P(F) = \frac{1}{3} \cdot 0.6 = 0.2$. Hence you can fill in the first piece



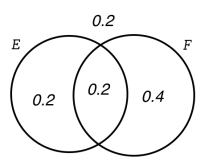
Since $P(F) = .6$ you get



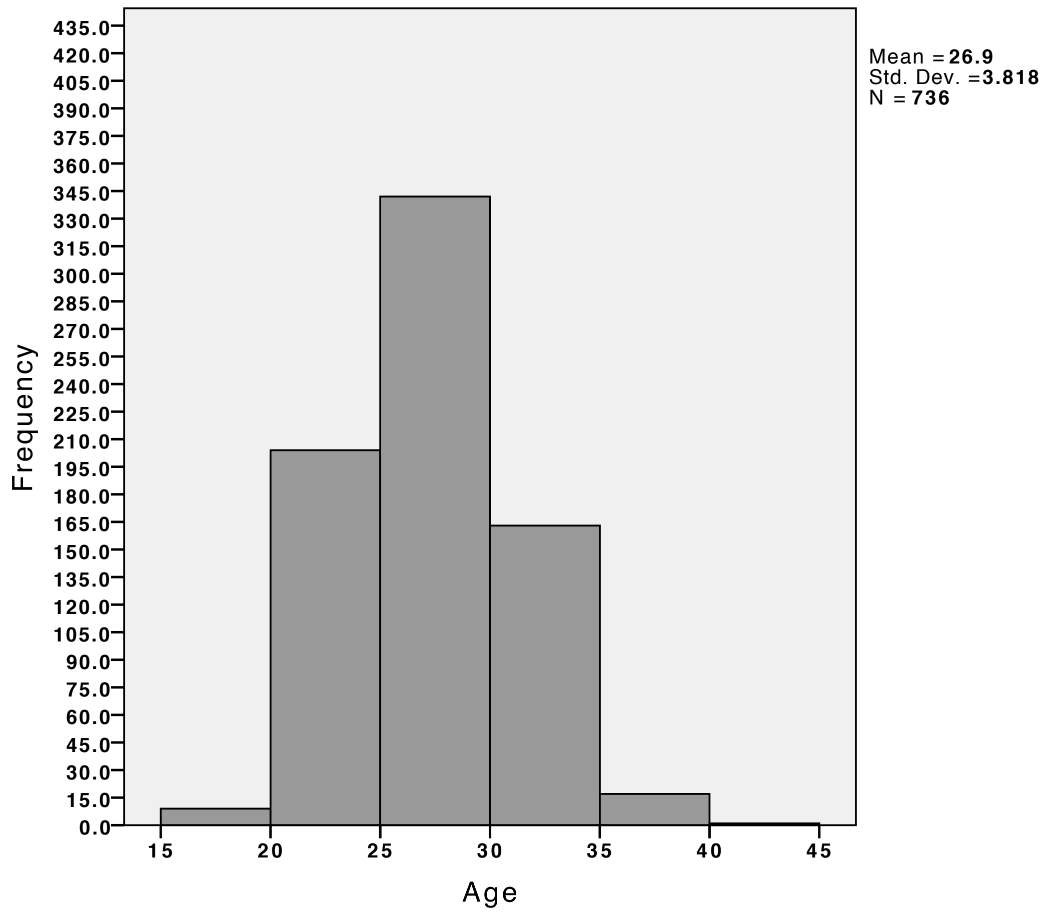
Since $P(E) = .4$ you get



Finally you finish with



9.(5pts) The histogram shown below, gives the frequency of age groups for all players in the World Cup of 2014. There were 736 players in the 2014 World Cup. (We assume that the histogram follows the convention that data at a boundary of categories goes in the category on the right.)



Which of the following statements can deduced from the information given in the histogram?

(a) More than one three quarters of the players were 25 years old or older.

(b) More than 150 players were exactly 32.5 years old

(c) At least one player was 15 years old.

(d) One player was 45 years old.

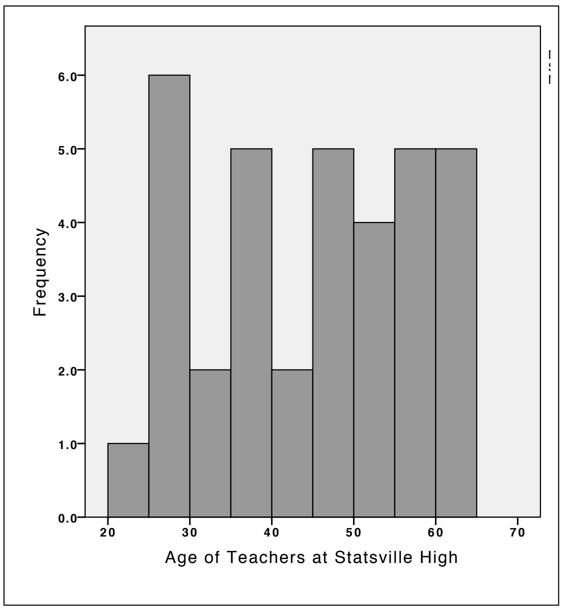
(e) At least 40% of the players were in the age bracket 25-30 (age in the interval $[25, 30)$).

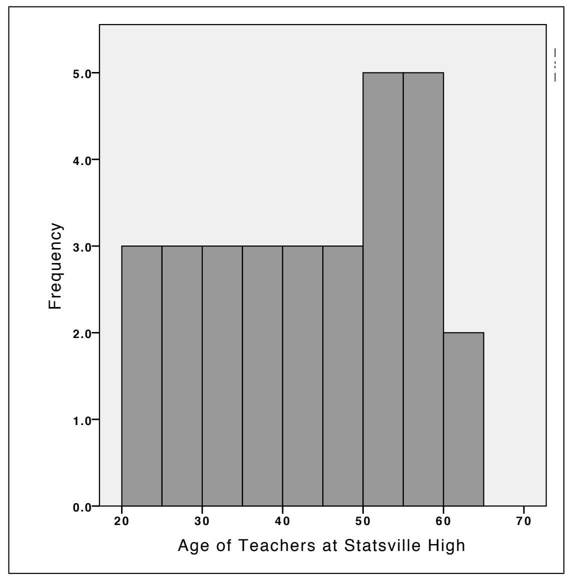
Solution. We cannot tell the precise ages of players in the given intervals, so we cannot conclude that there was a player who was 15 years old, 45 years old or 32.5 years old from the information given.
The number of players who were less than 25 years old was at least 195 (a conservative lower bound for the number in the class [20, 25)). Since one quarter of the players is $\frac{736}{4} = 184$, we see that more than one quarter of the players were younger than 25 and thus we cannot have that more than three quarters were 25 or older.
The number of players in the interval $[25, 30)$ is at least 330. Forty percent of the players is $0.4 \times 736 = 294.4$. Therefore at least 40% of the players were in the age bracket 25-30 .

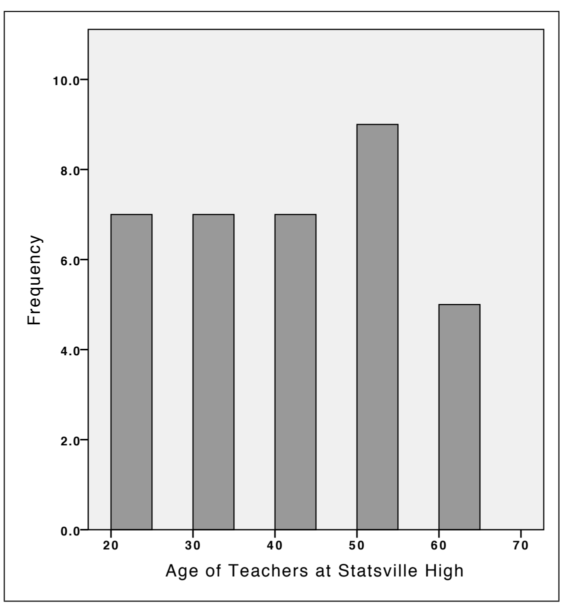
10.(5pts) The data given in the following stem and leaf plot shows the ages of all teachers at Statsville High School.

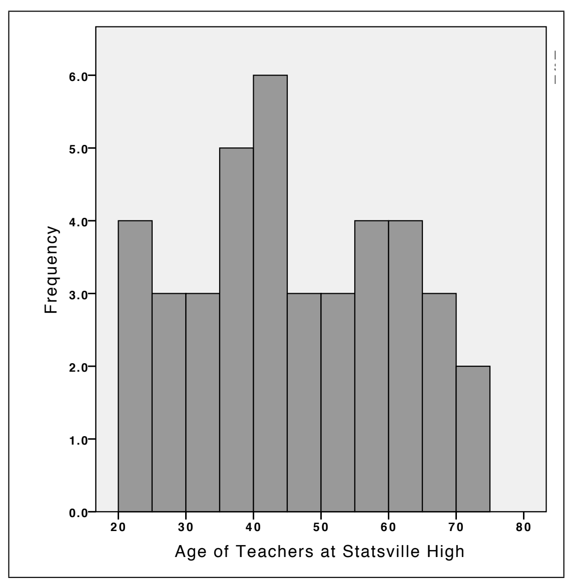
2	2	5	5	9	9	9	9		
3	0	0	5	5	7	7	9		
4	0	0	5	5	5	9	9		
5	0	0	1	4	6	7	8	8	9
6	0	1	2	3	4				

Which of the following gives a histogram of the data in the stem and leaf plot above.

(a) 

(b) 

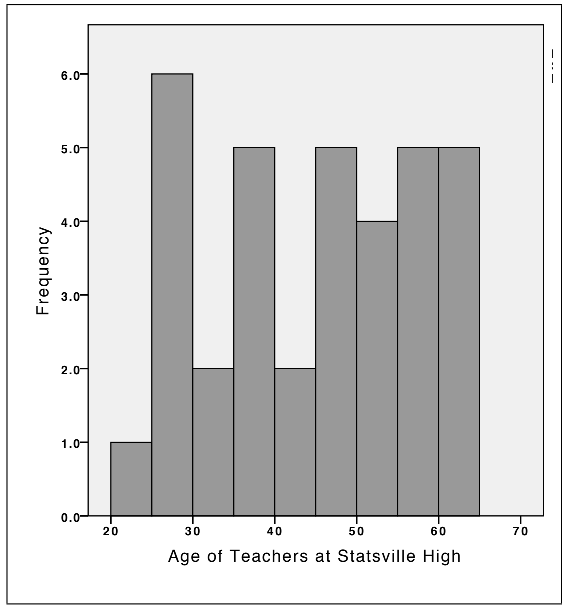
(c) 

(d) 

(e) None of the above

Solution. We see that if we choose our categories to be intervals of length 5, starting at 20, we get the following frequency distribution:

Mileage (Category)	**# of cars** (Frequency)
[20 , 25)	1
[25 , 30)	6
[30 , 35)	2
[35 , 40)	5
[40 , 45)	2
[45 , 50)	5
[50 , 55)	4
[55 , 60)	5
[60 , 65)	5



The only histogram which fits this data is the one shown on the right. It is conceivable that the creator of the histogram might not be following the convention to include data at the boundary in the interval on the right and instead might be using the intervals $(25, 30]$, $(30, 35]$, etc.... . However, we see that the related frequencies fit none of the histograms above even in the first two such intervals.

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Partial Credit

You must show your work on the partial credit problems to receive credit!

Where applicable, answers may be given in the form of products of numbers and symbols for factorials and numbers of permutations and combinations.

11.(12pts) Flip a fair coin 15 times and record the sequence of heads and tails. *Write your answers to the following questions using combinations of powers, permutations, combinations or factorials as appropriate.*

(a) What is the probability that we get exactly one head?

(b) What is the probability that we get exactly two heads?

(c) What is the probability that we get exactly five heads?

(d) What is the probability that we get at least 3 heads?

Solution. The number of ways that a 15 term sequence can have k heads is $C(15, k)$. Hence the probability that you have k heads in a randomly chosen sequence is $\frac{(C(15,k)}{2^{15}}$. So

(a) $\frac{C(15,1)}{2^{15}}$

(b) $\frac{C(15,2)}{2^{15}}$

(c) $\frac{C(15,5)}{2^{15}}$

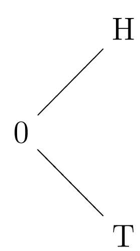
The probability of getting at least 3 heads is 1 minus the probability of getting 0, 1 or 2 heads:

$$1 - \left(\frac{C(15,0)}{2^{15}} + \frac{C(15,1)}{2^{15}} + \frac{C(15,2)}{2^{15}}\right)$$

12. (12pts) Imagine a 6 sided die but instead of the numbers from 1 to 6 on the faces, only the numbers 1 and 2 appear on the faces: 1 occurs twice and 2 occurs 4 times. Hence if you roll one of these die and record the number on top the probability that you get a 1 is $\frac{1}{3}$ and the probability that you get a 2 is $\frac{2}{3}$.

Consider the following game. First you flip a fair coin. If you get a head, you roll one of the die discussed in the last paragraph and then you roll it again. If you get a tail you roll only once.

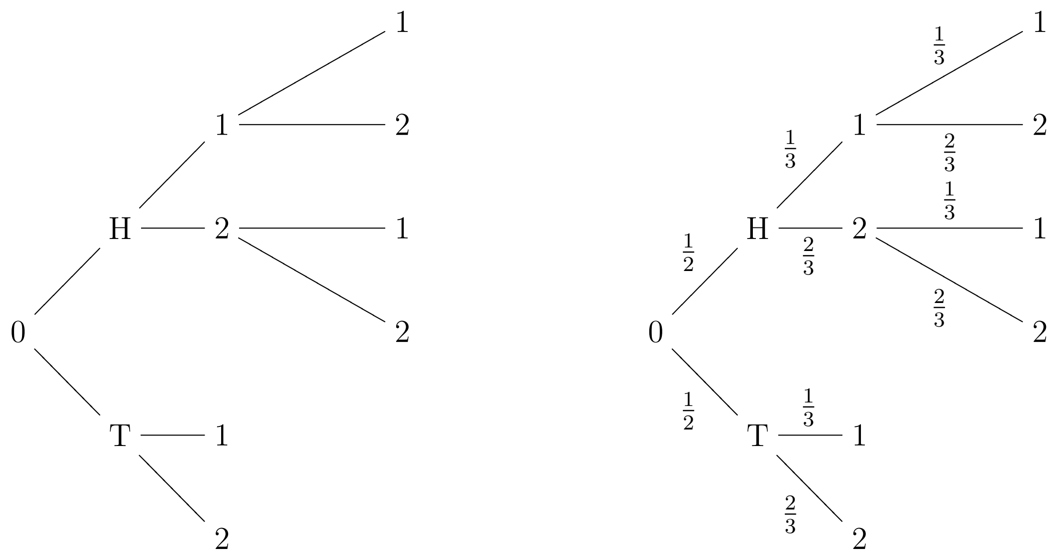
(a) Draw a tree diagram for this game and fill in the probabilities. Let H denote the event that you flipped a head and let T denote the event that you flipped a tail. Let 1 denote the event that you rolled a 1 and 2 denote the event that you rolled a 2. The first step in the diagram is given below.



(b) Using your tree diagram calculate the probability that you get exactly one 2 when you play this game once.

13. Initials: ____________

Solution.



The paths with one 2 are

(1) H 1 2
(2) H 2 1
(3) T 2

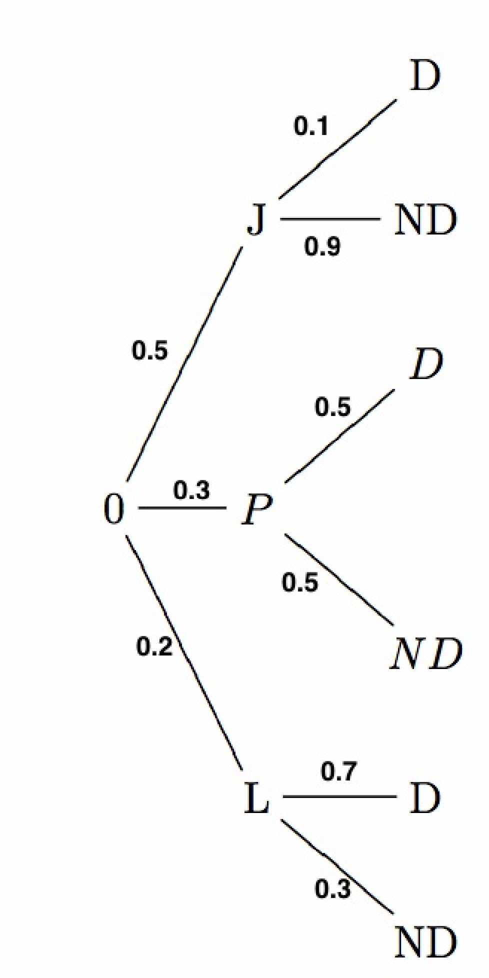
with probabilities

(1) $\frac{1}{2} \cdot \frac{1}{3} \cdot \frac{2}{3} = \frac{2}{18}$
(2) $\frac{1}{2} \cdot \frac{2}{3} \cdot \frac{1}{3} = \frac{2}{18}$
(3) $\frac{1}{2} \cdot \frac{2}{3} = \frac{2}{6} = \frac{6}{18}$

so the probability is $\frac{2+2+6}{18} = \frac{10}{18}$.

13.(12pts) John, Paul and Luke are responsible for the output of decanters at a glass factory. The table below shows the proportion of the output for which each is responsible and the probability that a decanter chosen at random from their respective outputs is defective. If a decanter shipped to Notre Dame bookstore is defective, what is the probability it was produced by John?

Worker	Proportion of Output	Pr(defective)
John	0.5	0.1
Paul	0.3	0.5
Luke	0.2	0.7



The tree diagram on the left shows the probabilities that the vase came from John (J), Paul (P) or Luke(L), and the probability that it was defective (D) in each case. We have that

$$P(J|D) =$$

$$\frac{P(J \cap D)}{P(J \cap D) + P(P \cap D) + P(L \cap D)}$$

$$= \frac{0.05}{0.05 + 0.15 + 0.14}$$

$$= \frac{0.05}{0.34} = \frac{5}{34}.$$

14.(12pts) A random sample of 1000 people was chosen from the Population of Iseland. For each person in the sample, the eye color and age was recorded. The results are shown in the table below.

		Eye Color			
		Blue	Green	Brown	**Totals**
Age	65 or over	100	90	70	260
	40 - 64	60	120	60	240
	25 - 39	90	110	50	250
	0 - 24	115	110	25	250
	Totals	365	430	205	1000

The record for one of the people from the sample is chosen at random.

Let B be the event that the record chosen is that of someone with blue eyes, let S be the event that the record chosen is that of someone aged 65 or older.

(a) What is the probability that the record chosen is that of someone with Blue eyes, P(B)?

$$P(B) = \frac{n(B)}{n(\text{Sample})} = \frac{365}{1000}.$$

(b) What is the probability that the record chosen is that of someone who has blue eyes and is aged 65 or older, $P(B \cap S)$?

$$P(B \cap S) = \frac{n(B \cap S)}{n(\text{Sample})} = \frac{100}{1000}.$$

(c) What is $P(B \cup S)$, that is the probability that the record chosen is that of someone who has blue eyes or is aged 65 or over or both.

$$P(B \cup S) = P(B) + P(S) - P(B \cap S) = \frac{365}{1000} + P(S) - \frac{100}{1000}.$$
$$= \frac{365}{1000} + \frac{n(S)}{n(\text{Sample})} - \frac{100}{1000} = \frac{365}{1000} + \frac{260}{1000} - \frac{100}{1000}$$
$$= \frac{525}{1000}$$

(d) What is $P(B|S)$, that is the probability that the record chosen is that of someone with blue eyes, given that it is a record of someone aged 65 or older?

$$P(B|S) = \frac{P(B \cap S)}{P(S)} = \frac{\frac{100}{1000}}{\frac{260}{1000}} = \frac{100}{260}.$$

(e) Are B and S independent events? Justify your answer.
B and S are not independent because

$$\frac{100}{260}(\text{approx.}385) = P(B|S) \neq P(S) = \frac{260}{1000} = 0.26.$$

Solution.

15.(2pts) You will get this 2 points if your instructor can read your name easily on the front page of the exam and you mark the answer boxes with an X (as opposed to a circle or any other mark).